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## TRANSLATION

INTERACTION BETWEEN A SHOCK WAVE IN AN ELASTO-  
PLASTIC MEDIUM AND A NONRIGID WALL

By

G. M. Lyakhov and N. I. Polyakova

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INTERACTION BETWEEN A SHOCK WAVE IN AN ELASTO-PLASTIC  
MEDIUM AND A NONRIGID WALL

G. M. Lyakhov and N. I. Polyakova, Moscow

The interaction between a wall and a wave in an elasto-plastic medium with an alternating sign of curvature of the stress-strain dependence  $\sigma = \sigma(\varepsilon)$  was examined earlier [1]. The case of small stresses lying on the concave section of the diagram  $\sigma = \sigma(\varepsilon)$ , when the wave has no pressure jump at the front, has been investigated. Below, based on earlier studies [1, 2], we will give a solution to the problem of the interaction between a plane wave and a nonrigid wall or the boundary of media in the case of large stresses, corresponding to the section of the diagram  $\sigma(\varepsilon)$  which is convex relative to axis  $\varepsilon$ . Here the wave is a shock wave.

1. We will examine a model of a medium with nonlinear dependence  $\sigma = \sigma(\varepsilon)$ . With small values of  $\sigma$  we have  $d^2\sigma/d\varepsilon^2 < 0$  and with the large values,  $d^2\sigma/d\varepsilon^2 > 0$ . Compression and unloading when  $\sigma < \sigma_s$  occur elastically, i.e., according to one law, but when  $\sigma > \sigma_s$  they occur according to different laws. Secondary loading takes place according to the law of unloading up to the stress achieved on first compression. Such a model is applicable to the ground and certain solid media.

We will examine large stresses, therefore we can consider the elastic section as small and approximate the relation  $\sigma = \sigma(\varepsilon)$  under

load with two straight lines (Fig. 1).

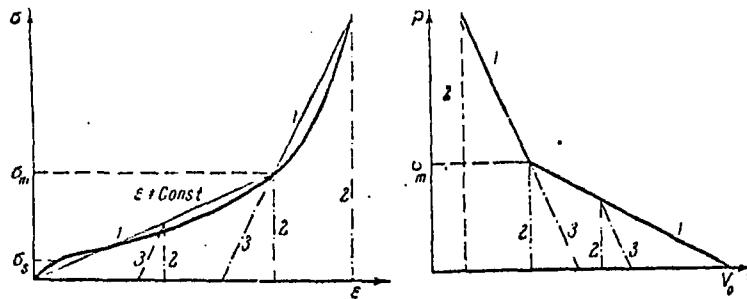


Fig. 1 a and b.

We will examine two variants of the approximation of the unloading curve. In the first case we will assume that unloading takes place at a constant residual deformation, i.e., along a line parallel to axis  $\sigma$ , in the second case the unloading line is parallel to the second section of the linearized compression diagram  $\sigma = \sigma(\epsilon)$ .

In Fig. 1a and b, broken line 1 corresponds to the loading line, straight lines 2 to the unloading lines under constant residual deformation, and straight line 3 to the unloading lines parallel to the second section of broken line 1.

Wave propagation in the first law of unloading has been examined by V. N. Rodionov, A. Ya. Sagomonyan, and other authors; the reflection from a rigid wall for the approximation of compression curve  $\sigma = \sigma(\epsilon)$  by a straight line was examined by S. Kaliski and Ya. Osieski [3].

We will change from parameters  $\sigma$  and  $\epsilon$  to the parameters, pressure  $p$  and volume  $V$ .

For the propagation of a plane wave in a medium unbounded in a direction perpendicular to its movement, or in a rod bounded by an incompressible shell, the compression of the medium corresponds to a

uniaxial deformed state. Thus

$$\sigma = -p, \quad \epsilon = (V - V_0) / V_0 \quad (1.1)$$

By virtue of (1.1), the relation  $p = p(V)$  will be linear if the relation  $\sigma = \sigma(\epsilon)$  is linear.

The equations of the factors approximating the compression curve, in a system of units  $p$  and  $V$ , are

$$p = -A_1^2 V + B_1 \text{ when } p \leq p_m, \quad p = -A_2^2 V + B_2 \text{ when } p > p_m \quad (1.2)$$

The thermal energy losses on compression of the medium are determined by the area of the figure on the plane  $pV$  bounded by a straight line which connects, in the compression diagram  $p = p(V)$ , the points corresponding to pressure at the front and ahead of the front and to an unloading line. Therefore, by the first unloading law ( $\partial V / \partial t = 0$ ), the energy losses by wave motions are as high as possible with the given compression law  $p = p(V)$ .

2. We will examine the interaction between a wave and a wall when the unloading line is parallel to axis  $p$ . Let us assume that when  $t = 0$  in the initial cross section of the medium  $h = 0$ , the pressure jumps to  $p_m$ , and then drops according to the given law

$$p = f(t) \quad (2.1)$$

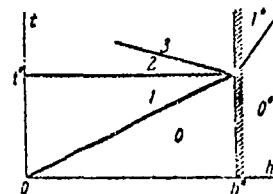


Fig. 2.

The shock wave begins to propagate throughout the medium. The flow of the shock wave behind the front, in Lagrangian coordinates where  $h$  is mass and  $t$  is time, is defined by the motion equations

$$\frac{\partial u}{\partial t} + \frac{\partial p}{\partial h} = 0, \quad \frac{\partial u}{\partial h} - \frac{\partial V}{\partial t} = 0 \quad (2.2)$$

Here  $u$  is the particle velocity.

In our case, unloading of the medium occurs behind the front (region 1 in Fig. 2) when  $\partial V/\partial t = 0$ , from where we obtain the solution of Eq. (2.2)

$$\frac{\partial u}{\partial h} = 0, \quad u = \varphi_1(t), \quad \frac{\partial p}{\partial h} = -\dot{\varphi}_1(t), \quad p = -h\dot{\varphi}_1(t) + \psi_1(t) \quad (2.3)$$

The functions  $\varphi_1(t)$  and  $\psi_1(t)$  must be determined from the initial and boundary conditions of the problem.

When  $h = 0$ , we have  $p = f(t)$ . Hence  $\psi_1(t) = f(t)$ . The relations at the front of the shock wave in the coordinates  $h, t$  have the form

$$p - p_0 = h^2 (V_0 - V), \quad u - u_0 = h (V_0 - V) \quad (2.4)$$

where  $h$  is the front velocity. We will assume the pressure and velocity of particles in front of the wave to be  $p_0 = 0$ ,  $u_0 = 0$ . Let the maximum pressure in the cross section  $h = 0$  correspond to maximum value  $p_m$  in the first part of the approximation. Then  $h = A_1$ . On the line of the front  $h = A_1 t$  we have

$$p = A_1 u, \quad f(t) - h\dot{\varphi}_1(t) - A_1\dot{\varphi}_1(t) = 0 \quad (2.5)$$

Integrating the last equation, we find  $\varphi_1(t)$ . In this way the solution in region 1 is obtained

$$p(h, t) = -h\dot{\varphi}_1(t) + f(t), \quad u = \varphi_1(t) \quad (2.6)$$

If the initial cross section  $p$  is given as

$$p(t) = p_m \left(1 - \frac{t}{b}\right) \quad (2.7)$$

then integrating (2.5), we find that in region 1

$$u - q_1(t) = \frac{p_m}{A_1} \left(1 - \frac{t}{2b}\right), \quad p = p_m \left(1 - \frac{t}{b} + \frac{h}{2A_1 b}\right) \quad (2.8)$$

Let us assume that a wall (or the boundary of media) is situated in cross section  $h = h^*$ . The relation  $p = p(V)$  in the second medium behind the wall is approximated by a straight line

$$p = -h^* V + p^* \quad (2.9)$$

When  $t^* = h^*/A_1$ , the front of the wave reaches the wall. Upon reflection form: region 2 - reflected shock wave, region 3 - reflected plastic wave, region 1\* - a passing wave behind the barrier. In region 2 loading of the medium occurs along the unloading line ( $V = V(h)$ ) up to the limit of elasticity which is different for different particles. The solution in region 2 relative to (2.3) is

$$p = -h\psi_2(t) + \psi_2(t), \quad u = q_2(t) \quad (2.10)$$

When  $h = 0$ , we have  $p = f(t)$ . Hence  $\psi_2(t) = f(t)$ . The front velocity is

$$h = \sqrt{\frac{p_2 - p_1}{V_1 - V_2}} \quad (2.11)$$

The velocity is determined by the angle of inclination of the secant in the diagram  $p = p(V)$ , drawn from a point where  $p_2$  corresponds to the pressure at the front to a point where  $p_1$  is the pressure ahead of the front. If the time of the wave action  $\theta$  in cross section  $h = 0$  is great, then the maximum pressure in the incident wave insignificantly decreases with the increase of  $h$ . The reflected wave moves over the region with little change of pressure. Therefore,

the front velocity of a reflected wave (when  $p > p_m$ ) can be assumed constant and, by virtue of our approximation, equals  $-A_2$ . Then the equation of the line of the front 2-3 (Fig. 2) is

$$h = -A_2 t + \frac{A_1 + A_2}{A_1} h^* \quad (2.12)$$

On this line in region 2 the pressure in each particle reaches the value which was at the front of the incident wave  $h = A_1 t$ . Thus by virtue of (2.10)

$$\begin{aligned} \dot{\varphi}_2(t) &= \dot{\varphi}_1(\lambda) + [f(t) - f(\lambda)] / A_1 \lambda \\ \lambda &= [(A_1 + A_2)t^* - A_2 t] / A_1 \end{aligned} \quad (2.13)$$

Integrating this equation, provided that  $\varphi_2(t^*) = \varphi_1(t^*)$ , we find  $\varphi_2(t)$  which together with the condition  $\psi_2(t) = f(t)$  determines the flow in region 2.

If when  $h = 0$  the pressure is defined by Eq. (2.7), then from (2.13) we will find that in region 2

$$\begin{aligned} \dot{\varphi}_2(t) &= \frac{p_m}{2A_1 t_0} \left( 1 - \frac{2t}{\lambda} \right) \\ \varphi_2(t) &= \frac{p_m}{2A_1} \left[ \frac{(A_1 + 2A_2)(t - t^*)}{A_1 t_0} + \frac{2(A_1 + A_2)h^*}{A_1 t_0} \ln \frac{\lambda}{t^*} - \frac{t^*}{t} + 2 \right] \end{aligned} \quad (2.14)$$

Unloading of a medium occurs in region 3, the solution has the form of (2.3).

The condition at the wall: the velocity of the particles of the medium, adjoining both sides of the wall, equals the wall velocity  $\varphi_3(t)$ . The pressure, acting from the side of the second medium when condition (2.9) is fulfilled, is related to the particle velocity by the ratio  $p = A^*u = A^*\varphi_3(t)$ .

Thus the equation of wall motion in region 3 is

$$m\dot{\varphi}_3(t) = -h^*\dot{\varphi}_3(t) + \psi_3(t) - A^*\varphi_3(t)$$

where  $m$  is the mass of the wall per unit cross-section area. Index 3 pertains to region 3. We find from the condition the function  $\psi_3(t)$  at the front of a reflected wave. According to (2.4) and (2.3)

$$p_3 - p_2 = -A_2(u_3 - u_2) = -h\dot{\varphi}_3(t) + h\dot{\varphi}_2(t) + \psi_3(t) - f(t) = A_2[\dot{\varphi}_2(t) - \dot{\varphi}_3(t)] \quad (2.15)$$

Having eliminated by means of (2.15) the function  $\psi_3$ , we will transform the equation of motion to

$$(A_2t - A_2t^* + m)\dot{\varphi}_3(t) = - (A_2 + A^*)\dot{\varphi}_3(t) + A_2\dot{\varphi}_2(t) - [(A_1 + A_2)t^* - A_2t]\dot{\varphi}_2(t) + f(t) \quad (2.16)$$

Integrating this equation, provided that  $\dot{\varphi}_3(t^*) = 0$ , we will find the velocity of the wall  $\varphi_3(t)$ . Having determined  $\psi_3(t)$  from (2.15), we obtain the solution in region 3. If the wall is rigid ( $m = \infty$ ), then in region 3 the particle velocity is equal to zero  $\dot{\varphi}_3(t) = 0$ . From (2.15) we will obtain

$$p(t) = \psi_3(t) = A_2\dot{\varphi}_2(t) + f(t) = \dot{\varphi}_2(t)A_2. \quad (2.17)$$

If condition (2.7) is fulfilled in cross section  $h = 0$ , then in region 3 the pressure is defined by the equation

$$p(t) = \frac{A_1 + A_2}{A_1} p_m \left( 1 + \frac{t}{t^*} - \frac{3}{2} \frac{t^*}{t} + \frac{A_1}{A_2} \frac{t^*}{t} \ln \frac{t}{t^*} \right) \quad (2.18)$$

Hence we obtain

$$p = \frac{A_1 + A_2}{A_1} p_m \left( 1 - \frac{t^*}{20} \right) \text{ when } t = t^*$$

In an incident wave at that instant of time according to (2.8)

$$p = p_m \left( 1 - \frac{t^*}{20} \right)$$

Hence the coefficient of reflection from the wall is

$$\eta = \frac{A_1 + A_2}{A_1} > 2 \quad (2.19)$$

If when  $h = 0$  the pressure is given in the form of (2.7) and the mass  $m$  is finite, then introducing a new variable  $\tau = t - t^*$ , we will obtain the differential equation of wall motion (2.16) in region 3 in the form

$$(m + A_2\tau) \dot{\varphi}_3(\tau) + (A_2 + A^*) \varphi_3(\tau) = p_m \frac{A_1 + A_2}{A_1} \left[ 1 + \frac{\tau}{6} - \frac{t^*}{20} + \frac{A_1 t^*}{A_2 0} \ln \left( 1 - \frac{A_2 \tau}{A_1 t^*} \right) \right]. \quad (2.20)$$

The solution of this equation, i.e., the expression for wall velocity in region 3, when  $A_1^* = A_1 = A_2/2$  is

$$\dot{\varphi}_3(\tau) = \frac{2p_m}{A_2} \left\{ \left( 1 - \frac{5t^*}{60} - \frac{2m}{5A_2 0} \right) (1 - \alpha^2) + \frac{3}{5} \frac{\tau}{6} + \frac{t^*}{20} \ln \left( 1 - \frac{2\tau}{t^*} \right) - \frac{t^*}{6} \left[ (1 - \alpha^2) \alpha \beta - \frac{1}{2} (\alpha \beta)^2 \ln \frac{V\sqrt{\alpha\beta} + 1}{(V\sqrt{\alpha\beta} - 1)(V\beta - 1)} \right] \right\} \quad (2.21)$$

where

$$\alpha = \frac{m}{m + A_2 \tau}, \quad \beta = 1 + \frac{h^*}{m},$$

For large values of  $t^*/\theta$  the pressure drop at the front of the incident wave in the section  $0 < h < h^*$  is great and the front velocity of a reflected plastic wave can be sharply distinguished from  $-A_2$ . In this case the flow in regions 2 and 3 must be obtained by the simultaneous determination of the boundary between them.

If the relation  $p(V)$  on compression is approximated by one straight line having an inclination  $A_1$  and unloading occurs along the vertical, then, assuming in Eq. (2.20)  $A^* = A_2 = A_1$ , we will obtain its solution as

$$\dot{\varphi}_3(\tau) = \frac{p_m}{A_1} \left\{ \left( 1 - \frac{t^*}{20} - \frac{m}{A_1 0} \right) (1 - \alpha^2) + \frac{2}{3} \frac{m}{A_1 0} \left( \frac{1}{\alpha} - \alpha^2 \right) + \frac{t^*}{6} \left[ \frac{1}{2} \left[ \left( \alpha + \frac{\tau}{\beta} \right)^2 - \left( 1 + \frac{\alpha}{\beta} \right)^2 \right] + \left( 1 - \frac{\alpha^2}{\beta^2} \right) \ln \frac{1 - \alpha/\beta}{\alpha + \tau/\beta} \right] \right\}$$

Here

$$\alpha = \frac{m}{m + A_1 \tau}, \quad \beta = \frac{m}{m + A_1 t^*}$$

The front velocity of a reflected plastic wave is equal to  $-A_1$ .

Depending on the parameters of the incident wave, on the characteristics of the media on both sides of the wall, on the mass, and on the removal of a wall from the first cross section, various configurations of a reflected-wave system are possible (in particular, unloading regions can be replaced by loading regions, etc.).

3. We will examine the interaction between a wave and a wall in the case where the unloading line is not parallel to axis  $p$ . Let us assume in cross section  $h = 0$  that the pressure changes in conformity with (2.7). We will approximate the curve of compression of the medium by a straight line (1.2), and the unloading line curve by a straight line

$$p = -A_2^2 V + B_2 \quad (3.1)$$

where  $B_2$  depends on the value of maximum stress attained on compression;  $p_m$  corresponds to maximum pressure in the first section of the approximation. The solution of Eqs. (2.2) defining the flow behind the wavefront formed in the medium, for linear approximation of the relation  $p = p(V)$ , as was shown in [1] is

$$p = F_1(h - A_2 t) + F_2(h + A_2 t), \quad A_2 u = F_1(h - A_2 t) - F_2(h + A_2 t) \quad (3.2)$$

The functions  $F_1$  and  $F_2$  are defined by the initial and boundary conditions. The flow behind the wavefront (region 1 in Fig. 3), obtained from (3.2), according to [1], is defined by the equations

$$p = p_m \left( 1 - \frac{A_1^2 A_2}{2 A_1 A_2} \frac{h}{0} - \frac{t}{0} \right) \quad (3.3)$$
$$A_2 u = p_m \left( \frac{A_1}{A_1} + \frac{h}{A_2 0} - \frac{A_1^2 A_2}{2 A_1 A_2} \frac{t}{0} \right)$$

When  $t = h^X/A_1 = t^X$ , the front reaches the wall. In this case regions 2 and  $1^X$  are formed. As in part 2, we will assume the equatio

of the reflected wavefront in the form of (2.12) and the relation  $p = p(V)$  in the second medium behind the wall in the form of (2.9).

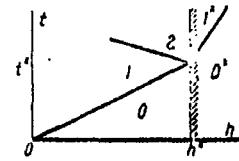


Fig. 3.

The region corresponding to region 2 in Fig. 2 in the case considered does not appear, since the velocity of the reflected wavefront coincides with the unloading velocity. The condition at the boundary of 1-2 (Fig. 3)

$$p_2 + A_2 u_2 = p_1 + A_1 u_1 \quad (3.4)$$

yields that the function

$$P_1(h - A_2 t) = p_m \left\{ \frac{A_1 + A_2}{2A_1} + \frac{(A_1 + A_2)^2}{4A_1 A_2 v_0} (h - A_2 t) \right\} \quad (3.5)$$

transfers from region 1 to region 2.

Thus the differential equation of wall motion is

$$m\dot{\varphi}(t) = p_2 - p_1^* = 2F_1(h^* - A_2 t) - (A_2 + A_1^*) \varphi(t)$$

or

$$\dot{\varphi} + C\varphi + Bt + D = 0 \quad (3.6)$$

where

$$C = \frac{A_1 + A_1^*}{m}, \quad B = \frac{(A_1 + A_2)^2}{2A_1 A_2} \frac{p_m}{0m}, \quad D = -p_m \frac{A_1 + A_2}{m A_1} \left[ 1 + \frac{(A_1 + A_2) h^*}{2A_2 v} \right]$$

Integrating (3.6), provided that condition  $\varphi(t^*) = 0$ , we will find the wall velocity

$$\varphi(t) = \frac{B}{C} \left\{ -t + \frac{h^*}{A_1} + \left( \frac{1}{C} - \frac{D}{B} - \frac{h^*}{A_1} \right) \left( 1 - \exp \left[ -C \left( t - \frac{h^*}{A_1} \right) \right] \right) \right\} \quad (3.7)$$

We will determine the second function in region 2 from the condition

$$F_1(h^* - A_2 t) = F_2(h^* + A_2 t) := A_2 p(t)$$

Hence

$$F_2(h + A_2 t) = \frac{A_1 + A_2}{2A_1} p_m \left\{ 1 + \frac{A_1 A_2}{2A_1^2 0} (2h^* - h + A_2 t) \right\} + A_2 \varphi \left( \frac{h - h^* + A_2 t}{A_2} \right).$$

The solution in region 2 is

$$p = \frac{A_1 + A_2}{A_1} p_m \left[ 1 + \frac{A_1 + A_2}{2A_1^2 0} h^* + \frac{A_1 + A_2}{2A_1^2} t \right] + A_2 \varphi \left( \frac{h - h^* + A_2 t}{A_2} \right)$$

$$u = \frac{A_1 + A_2}{2A_1 A_2^2} p_m \left( \frac{A_2 - h^*}{0} \right) + \varphi \left( \frac{h - h^* + A_2 t}{A_2} \right) \quad (3.8)$$

We obtain the solution in region 1<sup>X</sup> from the condition at the front of the passing wave  $p_1^X - A^X u_1^X = 2F_2^X = 0$  and from the condition that when  $h = h^X$  the particle velocity equals the wall velocity

$$p = A^X u = F_1^X (h - A^X t) + A^X \varphi \left( \frac{h^X - h + A^X t}{A^X} \right).$$

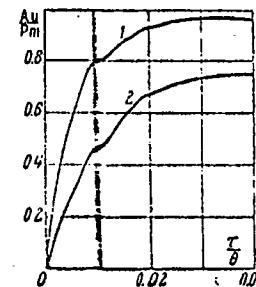


Fig. 4.

The wall velocity, defined by equation (3.7), first increases and then drops off. Differentiating (3.7) and equating the derivative to zero, we will find that the maximum value of velocity is reached at the instant of time  $\tau$ , determined from the condition

$$\exp \left[ -C \left( \tau - \frac{h^*}{A_1} \right) \right] - \frac{A_1 B}{A_1 B - C D - B C h^*} =$$

$$= \left[ 1 + \frac{A_1^2 + A_1}{A_1^2 + A_1} \frac{B}{B - C D} \right] \left[ 1 - \frac{A_1 (A_1^2 + A_1) h^*}{A_1^2 B} \right]^{-1} \quad (3.9)$$

If  $m = \infty$ , then in region 2 when  $h = h^X$

$$u = 0, \quad F_1(h^X - A_2 t) - F_2(h^X + A_2 t) = 0$$

Hence

$$\begin{aligned} F_2(h + A_2 t) &= F_1(2h^X - h - A_2 t) = p_m \frac{A_1 + A_2}{2A_1} \times \\ &\quad \times \left[ 1 + \frac{A_1 + A_2}{2A_2^2 0} (2h^X - h - A_2 t) \right] \\ p &= p_m \frac{A_1 + A_2}{A_1} \left[ 1 + \frac{A_1 + A_2}{2A_2^2 0} (h + A_2 t) \right], \quad u = \frac{p_m (A_1 + A_2)^2}{2A_1 A_2^2} (h - h^X) \end{aligned} \quad (3.10)$$

If  $m = 0$ , then from (3.6) we will find that when  $h = h^X$

$$\psi(t) = \frac{2F_1(h^X - A_2 t)}{A_1^2 + A_2^2} = \frac{(A_1 + A_2) p_m}{(A_1 + A_2) A_1} \left[ 1 + \frac{A_1 + A_2}{2A_2^2 0} (h^X - A_2 t) \right] \quad (3.11)$$

4. We will analyze the regularity of wall movement in an elementary case where the relation  $\sigma(\epsilon)$  on compression is approximated by one straight line.

We will examine the calculation results of the wall velocity, which are presented in Fig. 4 by two graphs corresponding to the different models of the medium.

In both cases the wall mass  $m$ , its distance from the initial cross section  $h^X$ , and the pressure change in cross section  $h = 0$  (Eq. (2.7)) are assumed identical.

The graphs correspond to the same law of compression of a medium (one linear factor), however, the unloading line is different: curve 1 is the unloading line coinciding with the load line; curve 2 is the unloading line parallel to axis  $p$ .

In the second case under these conditions, the energy loss on movement of the incident and reflected waves, as noted above, are maximum, and in the first case are minimum.

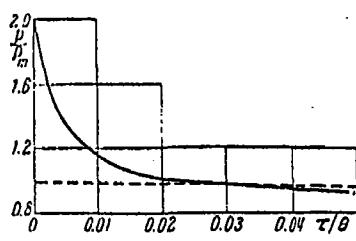


Fig. 5a.

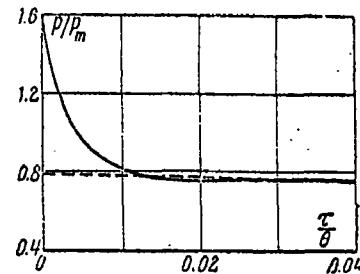


Fig. 5b.

The difference in the character of the graphs is due to the difference in energy loss. The greater the loss, the smaller the value of maximum wall velocity.

Figures 5a and 5b show the pressure change at the wall during its movement for these two models of the media: Fig. 5a is the medium governed by Hooke's law; Fig. 5b, the unloading line is vertical. In Fig. 5a and 5b the pressure in the wave in cross section  $h = h^X$  in the absence of a wall is shown by a dashed line. It is obvious from the figures that the presence of a wall appreciably distorts the wave only in a time interval small as compared with  $\theta$ .

Thus in an elasto-plastic medium with vertical unloading, just as in a linearly elastic medium governed by Hooke's law, even a wall with a relatively large mass is involved in motion together with the medium during a time interval small as compared to the time of the wave action.

This conclusion also pertains to media with an inclined unloading line.

REFERENCES

1. G. M. Lyakhov and N. I. Polyakova, Rasprostraneniye i vzaimodeystviye voln szhatiya i razrezheniya v uprugo-plasticheskikh sredakh. Izv. AN SSSR, OTN. Mekhanika i Mashinostroyeniye, No. 3, 1960.
2. G. M. Lyakhova, O vzaimodeystvii udarnykh voln v vodonasyshchennom grunte i v vode. Izv. AN SSSR, OTN, Mekhanika i Mashinostroyeniye, No. 1, 1961.
3. S. Kaliski and J. Osiecki, The Problem of Reflection by a Rigid or Elastic Wall of an Unloading Wave in a Body with Rigid Unloading Characteristic. Proc. of Vibr. Problems, Warsaw, No. 1, 1959.

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